

Test II Math 2255 3 March 2006

1. The position of an object at time t , as it traverses a circular path, is $\mathbf{r}(t) = b \cos \omega t \mathbf{i} + b \sin \omega t \mathbf{j}$. Here b, ω are both positive.

(a) (10 pts) Find the velocity $\mathbf{v}(t)$ and show it is orthogonal to $\mathbf{r}(t)$.

(b) (6 pts) Find the speed of the object. Show the calculation and simplify.

(c) (10 pts) Find the acceleration vector and show that its direction is always toward the center of the circle.

(d) (6 pts) Find and simplify the magnitude of the acceleration.

2. (12 pts) Find the unit tangent vector $\mathbf{T}(t)$ and find a set of parametric equations for the line tangent to the space curve given by $\mathbf{r}(t) = \langle 2 \sin t, 2 \cos t, 4 \sin^2 t \rangle$ at the point $(1, \sqrt{3}, 1)$.

3. (16 pts) Sketch, and if necessary, use words to describe the curve given by $\mathbf{r}(t) = \langle a \cos t, a \sin t, bt \rangle$. (Assume a and b are positive, $t \geq 0$.) What portion of the curve corresponds to the interval $t \in [0, 2\pi]$? Find the length of the curve over this interval.

4. (12 pts) Consider $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{xy}$. Find the limit along any line given by $x = at$, $y = bt$, a , b constants, not both zero. Can you conclude anything about the existence of $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{xy}$ from this information? Be precise.

5. (12 pts) Sketch a contour map for $f(x, y) = \sqrt{16 - x^2 - y^2}$.

6. (16 pts) Consider the curve given by $\mathbf{r}(t) = \langle 2 \sin t, 4 \sin t, 3 \cos t \rangle$. Find the curvature and torsion for the curve. You may use any means necessary but you must argue your answers.