

Test III Math 2255 April 7, 2006

1. (4 pts) Find the total differential of $f(x, y) = x \sec y$.
2. (8 pts) Use the Chain rule to find dw/dt where $w = \sqrt{x^2 + 3y^2}$, $x = \sin t$, $y = e^t$.
3. (10 pts) Find the gradient of $f(x, y) = y \cos(x - y)$ and the maximum value of the directional derivative at the point $P(0, \pi/3)$.
4. A metal plate has temperature at point (x, y) given by $T(x, y) = 50 - x^2 - 2y^2$.
 - (a) (16 pts) Find a parametric representation for the path of a heat-seeking particle placed at $P(4, 3)$ on the plate.
 - (b) (4 pts) Find the directions of no change in heat on the plate from the point $P(4, 3)$.

5. (6 pts) Is it true that when $D_{\mathbf{u}}f(x, y)$ exists, $D_{\mathbf{u}}f(x, y) = -D_{-\mathbf{u}}f(x, y)$? Explain your answer precisely.
6. (8 pts) Find an equation of the tangent plane to the surface given by $x^2 + y^2 - z^2 = 0$ at the point $P(5, 12, 13)$.
7. (12 pts) Find symmetric equations of the tangent to the curve of intersection of the surfaces $z = \sqrt{x^2 + y^2}$ and $2x + y + 2z = 20$ at the point $P(3, 4, 5)$. Find the cosine of the angle between the gradient vectors at this point.
8. (12 pts) Find the absolute extrema (give all three coordinates) for $f(x, y) = 2x - 2xy + y^2$ over the region in the xy -plane bounded by the graphs $y = x^2$ and $y = 1$.
9. (12 pts) Find and classify all local extrema for $f(x, y) = 120x - 120y - xy - x^2 - y^2$.

10. (12 pts) Use Lagrange multipliers to find the maximum value of e^{xy} given that $x^2 + y^2 = 8$. You may assume that x and y are both positive.