

FINAL EXAM Math 2255 May 2, 2006

1. (10 pts) Use parametric equations to describe the line that passes through the points $(7, -2, 3)$ and $(-1, 4, 5)$.
2. (10 pts) Find the distance from the point $(0, 2, 1)$ to the plane $3x - 2y + 6z = 1$.
3. (10 pts) Find an equation for the plane tangent to the surface in \mathbb{R}^3 given by $4x + 3y^2 + 2z^2 = 18$ at the point $(1, 2, 1)$.
4. (10 pts) Sketch the contour map (a collection of representative level sets) for the surface given by $f(x, y) = x^2 - 4y^2 - 1$. Label the contours and label intercepts.
5. (10 pts) Find $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$ or show that it does not exist.

6. (15 pts) Evaluate $\int_{x=0}^2 \int_{y=x}^2 x\sqrt{1+y^3} dy dx$ by switching the order of integration.
7. (15 pts) Find the velocity, speed, and acceleration of a particle whose position in space at time t is given by $\mathbf{r}(t) = \langle 2 \sin(t/2), 2 \cos(t/2), t \rangle$.
8. (20 pts) A heat seeking particle is on a plate at the point $(2, -3)$. The temperature at the point (x, y) on the plate is $T(x, y) = 20 - 4x^2 - y^2$. Find a parametric description of the path of the particle. Show all work.
9. (20 pts) Use Lagrange multipliers to find the point closest to the origin on the plane $x + y + z = 6$.

10. (20 pts) Examine the function $f(x, y) = -x^3 + 4xy - 2y^2 + 1$ for local extrema. Use the Second Partials Test as necessary.
11. (20 pts) Sketch the region in the xy -plane bounded by the parabola $y = 3x - x^2$ and the line $y = x$. Find the volume of the object with this base and top bounded by the plane $z = x + 4$.
12. (20 pts) Find the mass of the solid object bounded on the top by the paraboloid $z = r^2$, on the bottom by the plane $z = 0$, and on the sides by the cylinder $r = 1$. Assume the density at point (r, θ, z) is $\delta(r, \theta, z) = r$.

13. (20 pts) Use a triple integral and spherical coordinates to find the volume of the solid region bounded below by the upper half of the cone $x^2 + y^2 = z^2$ and above by the sphere $x^2 + y^2 + z^2 = 9$. (The solid looks roughly like a cone with a scoop of ice cream sitting in it.)